Approximating derivatives using least-squares best-fitting polynomials

1. A noisy sensor is reading speed at a rate of once every five seconds, and the reading is in meters per second. The readings are as follows:

0, 0, 0, 0, –0.35, 1.84, 1.56, –1.12, –4.70, 2.95, 3.77, 1.97, 5.81, 8.11, 10.62, 11.88, 17.45

Use the five-point approximation shown in the course slides:

1. For best-fitting least-squares linear polynomials:
   $a_1 = -0.2y_{n-4} - 0.1y_{n-3} + 0.1y_{n-1} + 0.2y_n$
   $a_0 = -0.2y_{n-4} + 0.2y_{n-2} + 0.4y_{n-1} + 0.6y_n$
2. For best-fitting least-squares linear polynomials:
   $a_2 = (\ 2y_{n-4} - y_{n-3} - 2y_{n-2} - y_{n-1} + 2y_n)/14$
   $a_1 = (26y_{n-4} - 27y_{n-3} - 40y_{n-2} - 13y_{n-1} + 54y_n)/70$
   $a_0 = (\ 3y_{n-4} - 5y_{n-3} - 3y_{n-2} + 9y_{n-1} + 31y_n)/35$

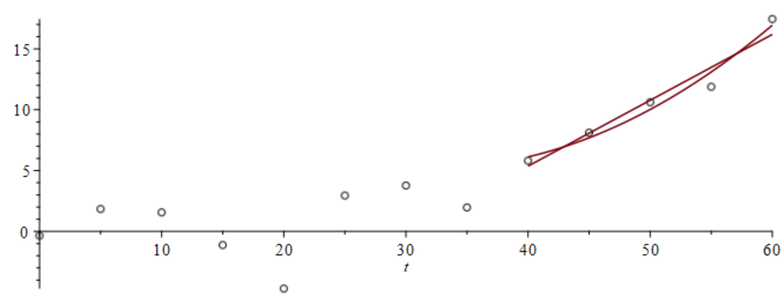
Use these to approximate the derivative at each point starting with the fifth.

Answer: Starting with the 5^{th} point, assuming all previous integrals are zero, and rounding to two decimal places for the quadratic polynomial:

–0.014, 0.0666, 0.0992, –0.0066, –0.2332, –0.0808, 0.1698, 0.293, 0.4008, 0.2472, 0.3968, 0.4926, 0.541
–0.0540, 0.2969, 0.2123, –0.4140, –1.0298, 0.7741, 1.2115, 0.1061, –0.1843, 0.7386, 0.8014, 0.2097, 0.8433

2. Plot the points with noise, and then plot the least-squares best-fitting polynomials that are used to estimate the derivatives at the last point assuming the first noisy signal was taken at time $t = 0$.

Answer:



3. With as much noise as was introduced into the data in Question 1, would it make more sense, or less sense, to use more points in finding the best-fitting least-squares polynomials?

Answer: The errors introduced into the data is quite significant, so more points would definitely give a much better approximation by eliminating some of that error.

4. Using the quadratic polynomial, estimate the derivative one time-step into the future.

Answer: $2a_2 + a_1 = 2 \times 0.377857142857143 + 4.216428571428570 = 4.972142857142856$